AN ENGINEERING METHOD FOR CALCULATING THE FLOW OF POLYMERS IN CHANNELS OF NONCIRCULAR CROSS SECTION

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On the basis of dimensional analysis, the article develops a method of generalizing data obtained when non-Newtonian media flow in channels of noncircular cross section. It proposes a method for the hydrodynamic calculation of these channels.

In many technological procedures for the processing of polymers — extrusion, shaping, pouring under pressure — use is made of shaping instruments in the form of channels of noncircular cross section [1, 2]. The rules governing the flow of the polymers in these channels are complicated, owing to the complex geometry of the shaping elements and the non-Newtonian character of the polymer flow. It is quite natural, therefore, that a great deal of attention has been devoted in the literature of this subject to channels of relatively simple geometry — annular, elliptical, rectangular, and triangular cross sections [2-5]. Nevertheless, even for these channels the theoretical calculation turns out to be rather complicated, so that it is feasible only for cases in which the liquid flowing is a Newtonian liquid or one which is subject to a specific rheological law. Therefore in practice, for the calculation of channels of noncircular cross section, use is sometimes made of nomograms constructed on the basis of an experimental investigation of the process of polymer flow in capillaries which have a ratio of cross-sectional dimensions equal to the ratio for the shaping instrument [6]. Miller [7] proposes a method for calculating the flow of polymers in channels with noncircular cross section which is based on introducing into the argument the variation of the average shearing stress as a function of the average shear rate of an empirical coefficient which is found from the flow of a Newtonian liquid in these channels and depends on the ratio of the cross-sectional dimensions. We propose below a method for calculating the flow of polymers in channels of noncircular cross section which is based on dimensional analysis. The method requires a knowledge only of how the shear rate $\dot{\gamma}$ varies with the shearing stress τ — that is to say, of the polymer flow curve.

The flow of polymers was studied on an experimental apparatus described in [5], which consists of a constant-pressure capillary viscosimeter. The geometric dimensions and the transverse cross section of the capillaries are shown in Table 1, from which it can be seen that the capillary cross sections selected for investigation are fairly complicated. The investigation was carried out on high-pressure polyethylene of brand No. 107-02-020 (MRTU 6-06-1085-69). The experiments were carried out at a temperature of 180°C.

In the investigation we also used experimental data obtained from the flow of a polystyrene melt [8], a 3% solution of carboxymethyl cellulose in water with 0.5% NaOH added, and an 8% solution of polyvinyl alcohol in water [9] in channels whose geometry is shown in Tables 2 and 3.

The experimental data obtained for the flow of the polyethylene melt in channels of various profiles (see Table 1) showed that the curves of flow rate versus pressure drop, when plotted on log—log paper, had the same shape, which was apparently determined solely by the physical properties of the polymer that characterized the flow process and was independent of the channel geometry. An objective physical

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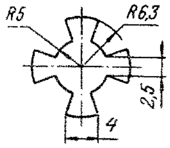
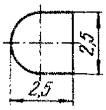
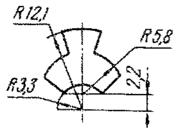
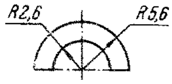
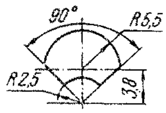
TABLE 1. Geometry of Capillaries

No. of item	Cross section	L, cm	S, cm^2	P, cm	R_e, cm
1		10	0,071	0,942	0,15
2		9,97	0,68	5,12	0,266
3		10	0,0562	0,894	0,126
4		10	0,52	3,48	0,298
5		9,96	0,386	3,174	0,2432
6		10,05	0,48	2,81	0,342

characteristic of the polymer under the conditions of flow is the function $\tau(\dot{\gamma})$. The channel geometry, on the other hand, affects the position of the flow-rate-versus-pressure-drop curve in the coordinate plane. This conclusion enables us, using dimension analysis, to propose a method for generalizing the data on the flow of polymers in channels of noncircular cross section.

TABLE 2. Geometry of Capillaries

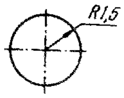
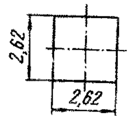
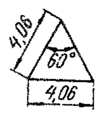
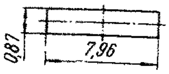
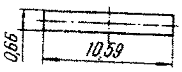
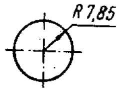
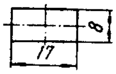
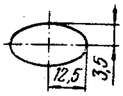
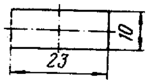
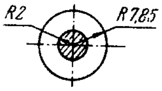
No. of item	Cross section	L, cm	S, cm^2	P, cm	R_e, cm
1		2	0,0705	0,942	0,15
2		2	0,0686	1,048	0,131
3		2	0,0712	1,218	0,117
4		2	0,0707	1,106	0,128
5		2	0,0693	1,766	0,0784
6		2	0,0697	2,25	0,062

TABLE 3. Geometry of Capillaries

No. of item	Cross section	L, cm	S, cm^2	P, cm	R_e, cm
1		362	1,935	4,93	0,785
2		336	1,36	5,0	0,543
3		300	1,375	5,45	0,505
4		344	2,3	6,6	0,697
5		362	1,81	6,19	0,585

We shall consider the laminar flow of a polymer under conditions of a steady-state pressure gradient along the length of the channel and a fully developed velocity profile, i.e., without taking account of inlet effects. We shall also assume that there is no slippage of the polymer with respect to the walls of the channel. As a characteristic of the cross section of the channel, we shall take the hydraulic or equivalent radius R_e, which is equal to the ratio of twice the area of the channel cross section, S, to the wetted perimeter, P. In this case, for a circular channel the hydraulic radius coincides with the radius of the channel. It should also be noted that the hydraulic radius so defined is widely used in hydrodynamic calculations for chemical engineering equipment in cases of Newtonian-liquid flow [10]. Then the pressure gradient in the general case for channels of arbitrary cross section with the flow of non-Newtonian liquids can be represented as a functional relationship

$$\frac{\Delta p}{L} = f[\eta(\dot{\gamma}),\ v,\ R_e], \tag{1}$$

where $\eta(\dot{\gamma})$ gives the effective viscosity η as a function of the shear rate $\dot{\gamma}$, and v is the average linear velocity of the polymer flow in the channel. If the above assumptions are made, the function (1) includes all the known parameters that affect the pressure gradient. Now, making use of dimensional analysis, from the function (1) we can easily obtain an equation which relates the parameters to one another in a unified formula:

$$\frac{\Delta p}{L} = c_1\eta(\dot{\gamma})\frac{v}{R_e^2} = c_1\eta(\dot{\gamma})\frac{Q}{SR_e^2}. \tag{2}$$

Equation (2) can be rewritten in the following form:

$$\frac{\Delta p R_e}{2L} = c\eta(\dot{\gamma})\frac{Q}{SR_e}, \tag{3}$$

where c is a quantity whose physical significance will be defined below.

As can be seen from (3), the left side has the dimensions of shearing stress and represents some reduced shearing stress which would be present at the wall of a channel of circular cross section having a radius equal to the hydraulic radius of the channel of noncircular cross section. The quantity Q/SR_e has the dimensions of shear rate and represents some reduced average shear rate $\dot{\gamma}_{av}$ which would be present in a channel of circular cross section having a radius equal to the hydraulic radius of the channel of noncircular cross section. Taking account of this, we can rewrite Eq. (3) in the following form:

$$\tau = c\eta(\dot{\gamma})\dot{\gamma}_{av}. \tag{4}$$

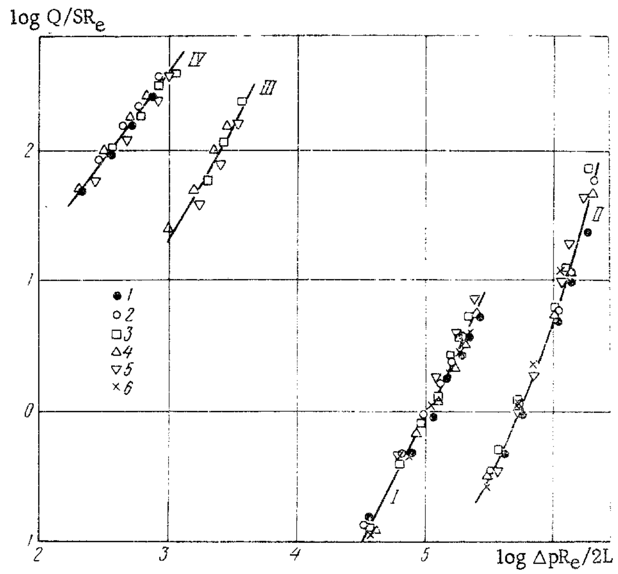


Fig. 1. Reduced curves of flow rate versus pressure drop for a polyethylene melt at a temperature of 180°C (curve I), a polystyrene melt [8] at a temperature of 190°C (II), a 3% solution of carboxymethyl cellulose in water [9] (III), and an 8% solution of polyvinyl alcohol in water [9] (IV) at a temperature of 22°C. The numbers next to the points correspond to the numbers of the capillaries in Tables 1-3. The units of log [ΔpR_e /2L] are dyn/cm^2, and those of log Q/SR_e are sec^{-1}.

From this it follows that

$$\eta(\dot{\gamma}) = \frac{\tau}{c\dot{\gamma}_{av}} = \frac{\tau}{\dot{\gamma}}, \tag{5}$$

where $\dot{\gamma}$ is the true shear rate corresponding to the shearing stress τ. From (5) it can be seen that the quantity c is the Rabinovich correction [2] for the calculation of the true shear rate at the wall of a capillary of circular cross section from the known average shear rate:

$$c = 3 + \frac{\tau}{\dot{\gamma}_{av}} \frac{d\dot{\gamma}_{av}}{d\tau}. \tag{6}$$

For a Newtonian liquid c = 4, and in this case Eq. (3) for a circular capillary becomes the Poiseuille equation.

Now let us consider how the equations obtained above, (3) or (4), can be justified for the experimental data we obtained and those known from the literature [8, 9]. The results of this processing are shown in Fig. 1, from which we can see the validity of the approach developed here for the flow of the various polymer systems in channels of various profiles. The flow data of each of the polymer systems in the various channels fit onto a single curve which is independent of the cross-sectional profile of the channels and define how the average shear rate varies with the shearing stress at the wall for a circular channel.

The results given above enable us to draw some important conclusions. Firstly, the hydraulic radius uniquely characterizes the cross section of a channel of arbitrary cross section when either Newtonian or non-Newtonian liquids flow through it. Secondly, the shape of the flow-rate-versus-pressure-drop curve in channels of arbitrary cross section depends only on the flow curve of the medium, and its position in the plane is uniquely determined by the value of the hydraulic radius. Thus, the proposed method for representing data on the flow of various polymer systems in channels of arbitrary cross section is a general one. This enables us to propose a simple method for the hydrodynamic calculation of the process of flow of non-Newtonian media in channels of noncircular cross section, based on a knowledge only of the flow curve of the medium and the geometric dimensions of the channel.

Now let us show how to make use of this method for carrying out the appropriate calculations. Suppose that we are given the flow curve of a polymer, $\dot{\gamma} = f(\tau)$, and the geometric dimensions of the channel: the cross-sectional area S, the perimeter P, and the length L. We are required to find how the flow rate varies with the pressure drop when the pressure drop is known. The calculations are carried out as follows:

1) we determine the hydraulic radius by means of the formula $R_e = 2S/P$;

2) we calculate the value of the shearing stress by the formula $\tau = \Delta p R_e/2L$;

3) from the function $\dot{\gamma} = f(\tau)$ we determine the true shear rate $\dot{\gamma}$ corresponding to a given τ;

4) from the value we have found for $\dot{\gamma}$ we calculate the average velocity gradient $\dot{\gamma}_{av}$. From Eq. (5) it can be seen that $\dot{\gamma}_{av} = \dot{\gamma}/c$, where c, defined by formula (6), us unknown, and to determine it we must know the function $\dot{\gamma}_{av} = f(\tau)$. Therefore we shall consider the following equation, which is equivalent to the expression $\dot{\gamma}_{av} = \dot{\gamma}/c$:

$$\dot{\gamma}_{av} = \frac{1}{\tau^3} \int_0^{\tau} \tau^2 \dot{\gamma} d\tau = \frac{1}{\tau^3} \int_0^{\tau} \tau^2 f(\tau)\, d\tau. \qquad (7)$$

However, using (7) for the direct calculation of $\dot{\gamma}_{av}$ is cumbersome, owing to the difficulty of specifying the flow curve $\dot{\gamma} = f(\tau)$ in analytic form. We therefore subdivide the flow curve into segments within each of which it can be approximately described by a power function. Then $\dot{\gamma} = k\tau^n$, where k and n are the constants of the power function. Taking account of this, we obtain from (7) the equation $\gamma_{av} = (\dot{\gamma}/n + 3)$. From this we can easily calculate the value of $\dot{\gamma}_{av}$. Moreover, it can be seen that in the limiting case, for a sufficiently large number of subdivisions, we have $n = (d \ln \gamma / d \ln \tau)$;

5) from the known quantities $\dot{\gamma}_{av}$, S, and R_e, we find the flow rate $Q = \dot{\gamma}_{av} S R_e$. Thus, repeating the calculation for other values of the pressure drop Δp, we can obtain Q as a function of Δp. In exactly the same way, we can solve the inverse problem: knowing the values of the flow rate, determine the pressure drops corresponding to them.

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NOTATION

τ	is the shearing stress;
$\dot{\gamma}$	is the shear rate;
η	is the effective viscosity;
R_e	is the hydraulic radius;
S	is the cross-sectional area of channel;
P	is the perimeter of cross section of channel;
L	is the length of channel;
Δp	is the pressure drop in channel;
$\dot{\gamma}_{av}$	is the average shear rate;
c	is the Rabinovich's correction;
Q	is the polymer flow rate;
k, n	are the constants of the power function.

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